Online Appendices to "Direct and Indirect Effects based on Changes-in-Changes"

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Abstract: We propose a novel approach for causal mediation analysis based on changes-in-changes assumptions restricting unobserved heterogeneity over time. This allows disentangling the causal effect of a binary treatment on a continuous outcome into an indirect effect operating through a binary intermediate variable (called mediator) and a direct effect running via other causal mechanisms. We identify average and quantile direct and indirect effects for various subgroups under the condition that the outcome is monotonic in the unobserved heterogeneity and that the distribution of the latter does not change over time conditional on the treatment and the mediator. We also provide a simulation study and two empirical applications regarding a training programme evaluation and maternity leave reform.

Keywords: Direct effects, indirect effects, mediation analysis, changes-in-changes, causal mechanisms, treatment effects.

JEL classification: C21.

Sections:

A Proof of Theorem 1

A.1 Average direct effect under $\mathbf{d = 1}$ conditional on $\mathbf{D = 1}$ and $\mathbf{M(1) = 0}$

In the following, we prove that $\theta_1^{1,0}(1) = E[Y_1(1,0) - Y_1(0,0)|D = 1, M_i(1) = 0] = E[Y_1 - Q_{00}(Y_0)|D = 1, M = 0]$. Using the observational rule, we obtain $E[Y_1(1,0)|D = 1, M(1) = 0] = E[Y_1|D = 1, M = 0]$. Accordingly, we have to show that $E[Y_1(0,0)|D = 1, M(1) = 0] = E[Q_{00}(Y_0)|D = 1, M = 0]$ to finish the proof.

Denote the inverse of $h(d,m,t,u)$ by $h^{-1}(d,m,t;y)$, which exists because of the strict monotonicity required in Assumption 1. Under Assumptions 1 and 3a, the conditional potential outcome distribution function equals

$$\begin{aligned} F_{Y_t(d,0)|D=1,M=0}(y) &\overset{A1}{=} \Pr(h(d,m,t,U_t) \leq y|D = 1, M = 0, T = t), \\ &= \Pr(U_t \leq h^{-1}(d,m,t;y)|D = 1, M = 0, T = t), \\ &\overset{A3a}{=} \Pr(U_t \leq h^{-1}(d,m,t;y)|D = 1, M = 0), \\ &\overset{A3a}{=} \Pr(U_{t'} \leq h^{-1}(d,m,t;y)|D = 1, M = 0), \\ &= F_{U_{t'}|10}(h^{-1}(d,m,t;y)), \end{aligned} \tag{A.1}$$

for $d, t, t' \in \{0, 1\}$. We use these quantities in the following.

First, evaluating $F_{Y_1(0,0)|D=1,M=0}(y)$ at $h(0,0,1,u)$ gives

$$F_{Y_1(0,0)|D=1,M=0}(h(0,0,1,u)) = F_{U_t|10}(h^{-1}(0,0,1;h(0,0,1,u))) = F_{U_t|10}(u),$$

for any $t \in \{0,1\}$. Applying $F^{-1}_{Y_1(0,0)|D=1,M=0}(q)$ to both sides, we have

$$h(0,0,1,u) = F^{-1}_{Y_1(0,0)|D=1,M=0}(F_{U_t|10}(u)). \tag{A.2}$$

Second, for $F_{Y_0(0,0)|D=1,M=0}(y)$ we have

$$F^{-1}_{U_t|10}(F_{Y_0(0,0)|D=1,M=0}(y)) = h^{-1}(0,0,0;y). \tag{A.3}$$

Combining (A.2) and (A.3) yields,

$$h(0,0,1,h^{-1}(0,0,0;y)) = F^{-1}_{Y_1(0,0)|D=1,M=0} \circ F_{Y_0(0,0)|D=1,M=0}(y). \tag{A.4}$$

Note that $h(0,0,1,h^{-1}(0,0,0;y))$ maps the period 1 (potential) outcome of an cross-sectional observation unit with the outcome y in period 0 under non-treatment without the mediator. Accordingly, $E[F^{-1}_{Y_1(0,0)|D=1,M=0} \circ F_{Y_0(0,0)|D=1,M=0}(Y_0)|D=1,M=0] = E[Y_1(0,0)|D=1,M=0]$. We can identify $F_{Y_0(0,0)|D=1,M=0}(y)$ under Assumption 2, but we cannot identify $F_{Y_1(0,0)|D=1,M=0}(y)$. However, we show in the following that we can identify the overall quantile-quantile transform $F^{-1}_{Y_1(0,0)|D=1,M=0} \circ F_{Y_0(0,0)|D=1,M=0}(y)$ under the additional Assumption 3b.

Under Assumptions 1 and 3b, the conditional potential outcome distribution function equals

$$\begin{aligned} F_{Y_t(d,0)|D=0,M=0}(y) &\overset{A1}{=} \Pr(h(d,m,t,U_t) \le y|D=0,M=0,T=t), \\ &= \Pr(U_t \le h^{-1}(d,m,t;y)|D=0,M=0,T=t), \\ &\overset{A3b}{=} \Pr(U_t \le h^{-1}(d,m,t;y)|D=0,M=0), \\ &\overset{A3b}{=} \Pr(U_{t'} \le h^{-1}(d,m,t;y)|D=0,M=0), \\ &= F_{U_{t'}|00}(h^{-1}(d,m,t;y)), \end{aligned} \tag{A.5}$$

for $d, t, t' \in \{0,1\}$. We repeat similar steps as above. First, evaluating $F_{Y_1(0,0)|D=0,M=0}(y)$ at $h(0,0,1,u)$ gives

$$F_{Y_1(0,0)D=0,M=0}(h(0,0,1,u)) = F_{U_t|00}(h^{-1}(0,0,1;h(0,0,1,u))) = F_{U_t|00}(u),$$

for any $t \in \{0,1\}$. Applying $F^{-1}_{Y_1(0,0)|D=0,M=0}(q)$ to both sides, we have

$$h(0,0,1,u) = F^{-1}_{Y_1(0,0)|D=0,M=0}(F_{U_t|00}(u)). \tag{A.6}$$

Second, for $F_{Y_0(0,0)|D=0,M=0}(y)$ we have

$$F^{-1}_{U_t|00}(F_{Y_0(0,0)|D=0,M=0}(y)) = h^{-1}(0,0,0;y). \tag{A.7}$$

Combining (A.6) and (A.7) yields,

$$h(0,0,1,h^{-1}(0,0,0;y)) = F^{-1}_{Y_1(0,0)|D=0,M=0} \circ F_{Y_0(0,0)|D=0,M=0}(y). \tag{A.8}$$

The left sides of (A.4) and (A.8) are equal. In contrast to (A.4), (A.8) contains only distributions that can be identified from observable data. In particular, $F_{Y_t(0,0)|D=0,M=0}(y) = \Pr(Y_t(0,0) \leq y|D = 0, M = 0) = \Pr(Y_t \leq y|D = 0, M = 0)$. Accordingly, we can identify $F^{-1}_{Y_1(0,0)|D=1,M=0} \circ F_{Y_0(0,0)|D=1,M=0}(y)$ by $Q_{00}(y) \equiv F^{-1}_{Y_1|D=0,M=0} \circ F_{Y_0|D=0,M=0}(y)$.

Parsing Y_0 through $Q_{00}(\cdot)$ in the treated group without mediator gives

$$\begin{aligned}
&E[Q_{00}(Y_0)|D=1,M=0] \\
&\quad= E[F^{-1}_{Y_1|D=0,M=0} \circ F_{Y_0|D=0,M=0}(Y_0)|D=1,M=0], \\
&\quad= E[F^{-1}_{Y_1(0,0)|D=0,M=0} \circ F_{Y_0(0,0)|D=0,M=0}(Y_0(1,0))|D=1,M=0], \\
&\quad\overset{A1,A3b}{=} E[h(0,0,1,h^{-1}(0,0,0;Y_0(1,0)))|D=1,M=0], \\
&\quad\overset{A2}{=} E[h(0,0,1,h^{-1}(0,0,0;Y_0(0,0)))|D=1,M=0], \\
&\quad\overset{A1,A3a}{=} E[F^{-1}_{Y_1(0,0)|D=1,M=0} \circ F_{Y_0(0,0)|D=1,M=0}(Y_0(0,0))|D=1,M=0], \\
&\quad= E[Y_1(0,0)|D=1,M=0] = E[Y_1(0,0)|D=1,M(1)=0],
\end{aligned} \tag{A.9}$$

which has data support because of Assumption 4a.

A.2 Quantile direct effect under $\mathbf{d=1}$ conditional on $\mathbf{D=1}$ and $\mathbf{M(1)=0}$

In the following, we prove that

$$\theta_1^{1,0}(q,1) = F^{-1}_{Y_1(1,0)|D=1,M(1)=0}(q) - F^{-1}_{Y_1(0,0)|D=1,M(1)=0}(q),$$
$$= F^{-1}_{Y_1|D=1,M=0}(q) - F^{-1}_{Q_{00}(Y_0)|D=1,M=0}(q).$$

For this purpose, we have to show that

$$F_{Y_1(1,0)|D=1,M(1)=0}(y) = F_{Y_1|D=1,M=0}(y) \text{ and} \tag{A.10}$$
$$F_{Y_1(0,0)|D=1,M(1)=0}(y) = F_{Q_{00}(Y_0)|D=1,M=0}(y), \tag{A.11}$$

which is sufficient to show that the quantiles are also identified. We can show (A.10) using the observational rule $F_{Y_1(1,0)|D=1,M(1)=0}(y) = F_{Y_1|D=1,M=0}(y) = E[1\{Y_1 \leq y\}|D=1,M=0]$, with $1\{\cdot\}$ being the indicator function.

In analogy to (A.9), we obtain

$$\begin{aligned}
&F_{Q_{00}(Y_0)|D=1,M=0}(y)\\
&\quad= E[1\{Q_{00}(Y_0) \leq y\}|D=1,M=0],\\
&\quad= E[1\{F^{-1}_{Y_1|D=0,M=0} \circ F_{Y_0|D=0,M=0}(Y_0) \leq y\}|D=1,M=0],\\
&\quad= E[1\{Y_1(0,0) \leq y\}|D=1,M=0],\\
&\quad= F_{Y_1(0,0)|D=1,M(1)=0}(y),
\end{aligned} \tag{A.12}$$

which proves (A.11).

A.3 Average direct effect under $\mathbf{d = 0}$ conditional on $\mathbf{D = 0}$ and $\mathbf{M(0) = 0}$

In the following, we show that $\theta_1^{0,0}(0) = E[Y_1(1,0) - Y_1(0,0)|D = 0, M(0) = 0] = E[Q_{10}(Y_0) - Y_1|D = 0, M = 0]$. Using the observational rule, we obtain $E[Y_1(0,0)|D = 0, M(0) = 0] = E[Y_1|D = 0, M = 0]$. Accordingly, we have to show that $E[Y_1(1,0)|D = 0, M(0) = 0] = E[Q_{10}(Y_0)|D = 0, M = 0]$ to finish the proof.

First, we use (A.5) to evaluate $F_{Y_1(1,0)|D=0,M=0}(y)$ at $h(1,0,1,u)$

$$F_{Y_1(1,0)|D=0,M=0}(h(1,0,1,u)) = F_{U_t|10}(h^{-1}(1,0,1;h(1,0,1,u))) = F_{U_t|10}(u),$$

for any $t \in \{0,1\}$. Applying $F^{-1}_{Y_1(1,0)|D=0,M=0}(q)$ to both sides, we have

$$h(1,0,1,u) = F^{-1}_{Y_1(1,0)|D=0,M=0}(F_{U_t|10}(u)). \tag{A.13}$$

Second, for $F_{Y_0(1,0)|D=0,M=0}(y)$ we have

$$F^{-1}_{U_t|10}(F_{Y_0(1,0)|D=0,M=0}(y)) = h^{-1}(1,0,0;y), \tag{A.14}$$

using (A.5). Combining (A.13) and (A.14) yields,

$$h(1,0,1,h^{-1}(1,0,0;y)) = F^{-1}_{Y_1(1,0)|D=0,M=0} \circ F_{Y_0(1,0)|D=0,M=0}(y). \tag{A.15}$$

Note that $h(1,0,1,h^{-1}(1,0,0;y))$ maps the period 1 (potential) outcome of an cross-sectional observation unit with the outcome y in period 0 under treatment without the mediator. Accordingly, $E[F^{-1}_{Y_1(1,0)|D=0,M=0} \circ F_{Y_0(1,0)|D=0,M=0}(Y_0)|D = 0, M = 0] = E[Y_1(1,0)|D = 1, M = 0]$. We can identify $F_{Y_0(1,0)|D=0,M=0}(y)$ under Assumption 2, but we cannot identify $F_{Y_1(1,0)|D=0,M=0}(y)$. However, we show in the following that we can identify the overall quantile-quantile transform $F^{-1}_{Y_1(1,0)|D=0,M=0} \circ F_{Y_0(1,0)|D=0,M=0}(y)$ under the additional Assumption 3a.

First, we use (A.1) to evaluate $F_{Y_1(1,0)|D=1,M=0}(y)$ at $h(1,0,1,u)$

$$F_{Y_1(1,0)|D=10,M=0}(h(1,0,1,u)) = F_{U_t|10}(h^{-1}(1,0,1;h(1,0,1,u))) = F_{U_t|10}(u),$$

for any $t \in \{0,1\}$. Applying $F^{-1}_{Y_1(1,0)|D=1,M=0}(q)$ to both sides, we have

$$h(1,0,1,u) = F^{-1}_{Y_1(1,0)|D=1,M=0}(F_{U_t|10}(u)). \tag{A.16}$$

Second, for $F_{Y_0(1,0)|D=0,M=0}(y)$ we have

$$F^{-1}_{U_t|10}(F_{Y_0(1,0)|D=1,M=0}(y)) = h^{-1}(1,0,0;y), \tag{A.17}$$

using (A.1). Combining (A.16) and (A.17) yields,

$$h(1,0,1,h^{-1}(1,0,0;y)) = F^{-1}_{Y_1(1,0)|D=1,M=0} \circ F_{Y_0(1,0)|D=1,M=0}(y). \tag{A.18}$$

The left sides of (A.15) and (A.18) are equal. In contrast to (A.15), (A.18) contains only distributions that can be identified from observable data. In particular, $F_{Y_t(1,0)|D=1,M=0}(y) = \Pr(Y_t(1,0) \leq y|D = 1, M = 0) = \Pr(Y_t \leq y|D = 1, M = 0)$. Accordingly, we can identify $F^{-1}_{Y_1(1,0)|D=0,M=0} \circ F_{Y_0(1,0)|D=0,M=0}(y)$ by $Q_{10}(y) \equiv F^{-1}_{Y_1|D=1,M=0} \circ F_{Y_0|D=1,M=0}(y)$.

Parsing Y_0 through $Q_{10}(\cdot)$ in the non-treated group without mediator gives

$$\begin{aligned}
&E[Q_{10}(Y_0)|D = 0, M = 0] \\
&\quad = E[F^{-1}_{Y_1|D=1,M=0} \circ F_{Y_0|D=1,M=0}(Y_0)|D = 0, M = 0], \\
&\quad = E[F^{-1}_{Y_1(1,0)|D=1,M=0} \circ F_{Y_0(1,0)|D=1,M=0}(Y_0(0,0))|D = 0, M = 0], \\
&\quad \overset{A1,A3a}{=} E[h(1,0,1,h^{-1}(1,0,0;Y_0(0,0)))|D = 0, M = 0], \\
&\quad \overset{A2}{=} E[h(1,0,1,h^{-1}(1,0,0;Y_0(1,0)))|D = 1, M = 0], \\
&\quad \overset{A1,A3b}{=} E[F^{-1}_{Y_1(1,0)|D=0,M=0} \circ F_{Y_0(1,0)|D=0,M=0}(Y_0(1,0))|D = 0, M = 0], \\
&\quad = E[Y_1(1,0)|D = 0, M = 0] = E[Y_1(1,0)|D = 0, M(0) = 0],
\end{aligned} \tag{A.19}$$

which has data support because of Assumption 4b.

A.4 Quantile direct effect under $\mathbf{d = 0}$ conditional on $\mathbf{D = 0}$ and $\mathbf{M(0) = 0}$

In the following, we prove that

$$\begin{aligned}\theta_1^{0,0}(q,0) &= F^{-1}_{Y_1(1,0)|D=0,M(0)=0}(q) - F^{-1}_{Y_1(0,0)|D=0,M(0)=0}(q),\\ &= F^{-1}_{Q_{10}(Y_0)|D=0,M=0}(q) - F^{-1}_{Y_1|D=0,M=0}(q).\end{aligned}$$

For this purpose, we have to show that

$$F_{Y_1(1,0)|D=0,M(0)=0}(y) = F_{Q_{10}(Y_0)|D=0,M=0}(y) \text{ and} \tag{A.20}$$

$$F_{Y_1(0,0)|D=0,M(0)=0}(y) = F_{Y_1|D=0,M=0}(y), \tag{A.21}$$

which is sufficient to show that the quantiles are also identified. We can show (A.21) using the observational rule $F_{Y_1(0,0)|D=0,M(0)=0}(y) = F_{Y_1|D=0,M=0}(y) = E[1\{Y_1 \leq y\}|D = 0, M = 0]$.

Furthermore, in analogy to (A.19), we obtain

$$\begin{aligned}&F_{Q_{10}(Y_0)|D=0,M=0}(y)\\ &\quad= E[1\{Q_{10}(Y_0) \leq y\}|D = 0, M = 0],\\ &\quad= E[1\{F^{-1}_{Y_1|D=1,M=0} \circ F_{Y_0|D=1,M=0}(Y_0) \leq y\}|D = 0, M = 0],\\ &\quad= E[1\{Y_1(1,0) \leq y\}|D = 0, M = 0],\\ &\quad= F_{Y_1(1,0)|D=0,M(0)=0}(y),\end{aligned}$$

which proves (A.20).

A.5 Average direct effect under $\mathbf{d = 0}$ conditional on $\mathbf{D = 0}$ and $\mathbf{M(0) = 1}$

In the following, we show that $\theta_1^{0,1}(0) = E[Y_1(1,1) - Y_1(0,1)|D = 0, M(0) = 1] = E[Q_{11}(Y_0) - Y_1|D = 0, M = 1]$. Using the observational rule, we obtain $E[Y_1(0,1)|D = 0, M(0) = 1] = E[Y_1|D = 0, M = 1]$. Accordingly, we have to show that $E[Y_1(1,1)|D = 0, M(0) = 1] = E[Q_{11}(Y_0)|D = 0, M = 1]$ to finish the proof.

Under Assumptions 1 and 3c, the conditional potential outcome distribution function equals

$$\begin{aligned} F_{Y_t(d,0)|D=1,M=0}(y) &\overset{A1}{=} \Pr(h(d,m,t,U_t) \leq y|D = 0, M = 1, T = t), \\ &= \Pr(U_t \leq h^{-1}(d,m,t;y)|D = 0, M = 1, T = t), \\ &\overset{A3c}{=} \Pr(U_t \leq h^{-1}(d,m,t;y)|D = 0, M = 1), \\ &\overset{A3c}{=} \Pr(U_{t'} \leq h^{-1}(d,m,t;y)|D = 0, M = 1), \\ &= F_{U_{t'}|01}(h^{-1}(d,m,t;y)), \end{aligned} \tag{A.22}$$

for $d, t, t' \in \{0,1\}$. We use these quantities in the following.

First, evaluating $F_{Y_1(1,1)|D=0,M=1}(y)$ at $h(1,1,1,u)$ gives

$$F_{Y_1(1,1)|D=0,M=1}(h(1,1,1,u)) = F_{U_t|01}(h^{-1}(1,1,1;h(1,1,1,u))) = F_{U_t|01}(u),$$

for any $t \in \{0,1\}$. Applying $F^{-1}_{Y_1(1,1)|D=0,M=1}(q)$ to both sides, we have

$$h(1,1,1,u) = F^{-1}_{Y_1(1,1)|D=0,M=1}(F_{U_t|01}(u)). \tag{A.23}$$

Second, for $F_{Y_0(1,1)|D=0,M=1}(y)$ we have

$$F^{-1}_{U_t|01}(F_{Y_0(1,1)|D=0,M=1}(y)) = h^{-1}(1,1,0;y). \tag{A.24}$$

Combining (A.23) and (A.24) yields,

$$h(1,1,1,h^{-1}(1,1,0;y)) = F^{-1}_{Y_1(1,1)|D=0,M=1} \circ F_{Y_0(1,1)|D=0,M=1}(y). \tag{A.25}$$

Note that $h(1,1,1,h^{-1}(1,1,0;y))$ maps the period 1 (potential) outcome of an cross-sectional observation unit with the outcome y in period 0 under treatment with the mediator. Accordingly, $E[F^{-1}_{Y_1(1,1)|D=0,M=1} \circ F_{Y_0(1,1)|D=0,M=1}(Y_0)|D = 0, M = 1] = E[Y_1(1,1)|D = 0, M = 1]$. We can identify $F_{Y_0(1,1)|D=0,M=1}(y) = F_{Y_0|D=0,M=1}(y)$ under Assumption 2, but we cannot identify $F_{Y_1(1,1)|D=0,M=1}(y)$. We show in the following that we can identify the overall quantile-quantile transform $F^{-1}_{Y_1(1,1)|D=0,M=1} \circ F_{Y_0(1,1)|D=0,M=1}(y)$ under the additional Assumption 3d.

Under Assumptions 1 and 3d, the conditional potential outcome distribution function equals

$$\begin{aligned}
F_{Y_t(d,1)|D=1,M=1}(y) &\overset{A1}{=} \Pr(h(d,m,t,U_t) \leq y|D = 1, M = 1, T = t), \\
&= \Pr(U_t \leq h^{-1}(d,m,t;y)|D = 1, M = 1, T = t), \\
&\overset{A3d}{=} \Pr(U_t \leq h^{-1}(d,m,t;y)|D = 1, M = 1), \\
&\overset{A3d}{=} \Pr(U_{t'} \leq h^{-1}(d,m,t;y)|D = 1, M = 1), \\
&= F_{U_{t'}|11}(h^{-1}(d,m,t;y)),
\end{aligned} \tag{A.26}$$

for $d, t, t' \in \{0,1\}$. We repeat similar steps as above. First, evaluating $F_{Y_1(1,1)|D=1,M=1}(y)$ at $h(1,1,1,u)$ gives

$$F_{Y_1(1,1)|D=1,M=1}(h(1,1,1,u)) = F_{U_t|11}(h^{-1}(1,1,1;h(1,1,1,u))) = F_{U_t|11}(u),$$

for any $t \in \{0,1\}$. Applying $F^{-1}_{Y_1(1,1)|D=1,M=1}(q)$ to both sides, we have

$$h(1,1,1,u) = F^{-1}_{Y_1(1,1)|D=1,M=1}(F_{U_t|11}(u)). \tag{A.27}$$

Second, for $F_{Y_0(1,1)|D=1,M=1}(y)$ we have

$$F^{-1}_{U_t|11}(F_{Y_0(1,1)|D=1,M=1}(y)) = h^{-1}(1,1,1;y). \tag{A.28}$$

Combining (A.27) and (A.28) yields,

$$h(1,1,1,h^{-1}(1,1,0;y)) = F^{-1}_{Y_1(1,1)|D=1,M=1} \circ F_{Y_0(1,1)|D=1,M=1}(y). \tag{A.29}$$

The left sides of (A.25) and (A.29) are equal. In contrast to (A.25), (A.29) contains only distributions that can be identified from observable data. In particular, $F_{Y_t(1,1)|D=1,M=1}(y) = \Pr(Y_t(1,1) \leq y|D=1,M=1) = \Pr(Y_t \leq y|D=1,M=1)$. Accordingly, we can identify $F^{-1}_{Y_1(1,1)|D=0,M=1} \circ F_{Y_0(1,1)|D=0,M=1}(y)$ by $Q_{11}(y) \equiv F^{-1}_{Y_1|D=1,M=1} \circ F_{Y_0|D=1,M=1}(y)$.

Parsing Y_0 through $Q_{11}(\cdot)$ in the non-treated group with mediator gives

$$\begin{aligned}
&E[Q_{11}(Y_0)|D=0,M=1] \\
&\quad = E[F^{-1}_{Y_1|D=1,M=1} \circ F_{Y_0|D=1,M=1}(Y_0)|D=0,M=1], \\
&\quad = E[F^{-1}_{Y_1(1,1)|D=1,M=1} \circ F_{Y_0(1,1)|D=1,M=1}(Y_0(0,1))|D=0,M=1], \\
&\quad \overset{A1,A3d}{=} E[h(1,1,1,h^{-1}(1,1,0;Y_0(0,1)))|D=0,M=1], \\
&\quad \overset{A2}{=} E[h(1,1,1,h^{-1}(1,1,0;Y_0(0,0)))|D=0,M=1], \\
&\quad \overset{A1,A3c}{=} E[F^{-1}_{Y_1(1,1)|D=0,M=1} \circ F_{Y_0(1,1)|D=0,M=1}(Y_0(0,0))|D=0,M=1], \\
&\quad = E[Y_1(1,1)|D=0,M=1] = E[Y_1(1,1)|D=0,M(0)=1],
\end{aligned} \tag{A.30}$$

which has data support because of Assumption 4c.

A.6 Quantile direct effect under $\mathbf{d=0}$ conditional on $\mathbf{D=0}$ and $\mathbf{M(0)=1}$

In the following, we show that

$$\begin{aligned}
\theta_1^{0,1}(q,0) &= F^{-1}_{Y_1(1,1)|D=0,M(0)=1}(q) - F^{-1}_{Y_1(0,1)|D=0,M(0)=1}(q), \\
&= F^{-1}_{Q_{11}(Y_0)|D=0,M=1}(q) - F^{-1}_{Y_1|D=0,M=1}(q).
\end{aligned}$$

For this purpose, we have to prove that

$$F_{Y_1(1,1)|D=0,M(0)=1}(y) = F_{Q_{11}(Y_0)|D=0,M=1}(y) \text{ and} \tag{A.31}$$

$$F_{Y_1(0,1)|D=0,M(0)=1}(y) = F_{Y_1|D=0,M=1}(y), \tag{A.32}$$

which is sufficient to show that the quantiles are also identified. We can show (A.32) using the observational rule $F_{Y_1(0,1)|D=0,M(0)=1}(y) = F_{Y_1|D=0,M=1}(y) = E[1\{Y_1 \leq y\}|D = 0, M = 1]$.

In analogy to (A.30), we obtain

$$\begin{aligned}
&F_{Q_{11}(Y_0)|D=0,M=1}(y)\\
&\quad = E[1\{Q_{11}(Y_0) \leq y\}|D = 0, M = 1],\\
&\quad = E[1\{F^{-1}_{Y_1|D=1,M=1} \circ F_{Y_0|D=1,M=1}(Y_0) \leq y\}|D = 0, M = 1],\\
&\quad = E[1\{Y_1(1, 1) \leq y\}|D = 0, M = 0],\\
&\quad = F_{Y_1(1,1)|D=0,M(0)=1}(y),
\end{aligned} \tag{A.33}$$

which proves (A.31).

A.7 Average direct effect under $\mathbf{d = 1}$ conditional on $\mathbf{D = 1}$ and $\mathbf{M(1) = 1}$

In the following, we show that $\theta_1^{1,1}(1) = E[Y_1(1, 1) - Y_1(0, 1)|D = 1, M(1) = 1] = E[Y_1 - Q_{01}(Y_0)|D = 1, M = 1]$. Using the observational rule, we obtain $E[Y_1(1, 1)|D = 1, M(1) = 1] = E[Y_1|D = 1, M = 1]$. Accordingly, we have to show that $E[Y_1(0, 1)|D = 1, M(1) = 1] = E[Q_{01}(Y_0)|D = 1, M = 1]$ to finish the proof.

First, using (A.26) to evaluate $F_{Y_1(0,1)|D=1,M=1}(y)$ at $h(0, 1, 1, u)$ gives

$$F_{Y_1(0,1)|D=1,M=1}(h(0, 1, 1, u)) = F_{U_t|11}(h^{-1}(0, 1, 1; h(0, 1, 1, u))) = F_{U_t|11}(u),$$

for any $t \in \{0,1\}$. Applying $F^{-1}_{Y_1(0,1)|D=1,M=1}(q)$ to both sides, we have

$$h(0,1,1,u) = F^{-1}_{Y_1(0,1)|D=1,M=1}(F_{U_t|11}(u)). \tag{A.34}$$

Second, for $F_{Y_0(0,1)|D=0,M=1}(y)$ we obtain

$$F^{-1}_{U_t|11}(F_{Y_0(0,1)|D=1,M=1}(y)) = h^{-1}(0,1,0;y), \tag{A.35}$$

using (A.26). Combining (A.34) and (A.35) yields,

$$h(0,1,1,h^{-1}(0,1,0;y)) = F^{-1}_{Y_1(0,1)|D=1,M=1} \circ F_{Y_0(0,1)|D=1,M=1}(y). \tag{A.36}$$

Note that $h(0,1,1,h^{-1}(0,1,0;y))$ maps the period 1 (potential) outcome of an cross-sectional observation unit with the outcome y in period 0 under non-treatment with the mediator. Accordingly, $E[F^{-1}_{Y_1(1,1)|D=0,M=1} \circ F_{Y_0(1,1)|D=0,M=1}(Y_0)|D=0, M=1] = E[Y_1(1,1)|D=0, M=1]$. We can identify $F_{Y_0(1,1)|D=0,M=1}(y) = F_{Y_0|D=0,M=1}(y)$ under Assumption 2, but we cannot identify $F_{Y_1(1,1)|D=0,M=1}(y)$. We show in the following that we can identify the overall quantile-quantile transform $F^{-1}_{Y_1(1,1)|D=0,M=1} \circ F_{Y_0(1,1)|D=0,M=1}(y)$ under the additional Assumption 3c.

First, using (A.22) to evaluate $F_{Y_1(0,1)|D=0,M=1}(y)$ at $h(0,1,1,u)$ gives

$$F_{Y_1(0,1)|D=0,M=1}(h(0,1,1,u)) = F_{U_t|01}(h^{-1}(0,1,1;h(0,1,1,u))) = F_{U_t|01}(u),$$

for any $t \in \{0,1\}$. Applying $F^{-1}_{Y_1(0,1)|D=0,M=1}(q)$ to both sides, we have

$$h(0,1,1,u) = F^{-1}_{Y_1(0,1)|D=0,M=1}(F_{U_t|01}(u)). \tag{A.37}$$

Second, for $F_{Y_0(0,1)|D=0,M=1}(y)$ we obtain

$$F^{-1}_{U_t|01}(F_{Y_0(0,1)|D=0,M=1}(y)) = h^{-1}(0,1,1;y), \tag{A.38}$$

using (A.22). Combining (A.37) and (A.38) yields,

$$h(0,1,1,h^{-1}(0,1,0;y)) = F^{-1}_{Y_1(0,1)|D=0,M=1} \circ F_{Y_0(0,1)|D=0,M=1}(y). \tag{A.39}$$

The left sides of (A.36) and (A.39) are equal. In contrast to (A.36), (A.39) contains only distributions that can be identified from observable data. In particular, $F_{Y_t(0,1)|D=0,M=1}(y) = \Pr(Y_t(0,1) \leq y|D=0,M=1) = \Pr(Y_t \leq y|D=0,M=1)$. Accordingly, we can identify $F^{-1}_{Y_1(0,1)|D=1,M=1} \circ F_{Y_0(0,1)|D=1,M=1}(y)$ by $Q_{01}(y) \equiv F^{-1}_{Y_1|D=0,M=1} \circ F_{Y_0|D=0,M=1}(y)$.

Parsing Y_0 through $Q_{01}(\cdot)$ in the treated group with mediator gives

$$\begin{aligned}
&E[Q_{01}(Y_0)|D=1,M=1] \\
&\quad= E[F^{-1}_{Y_1|D=0,M=1} \circ F_{Y_0|D=0,M=1}(Y_0)|D=1,M=1], \\
&\quad= E[F^{-1}_{Y_1(0,1)|D=0,M=1} \circ F_{Y_0(0,1)|D=0,M=1}(Y_0(1,1))|D=1,M=1], \\
&\quad\overset{A1,A3c}{=} E[h(0,1,1,h^{-1}(0,1,0;Y_0(1,1)))|D=1,M=1], \\
&\quad\overset{A2}{=} E[h(0,1,1,h^{-1}(0,1,0;Y_0(0,1)))|D=1,M=1], \\
&\quad\overset{A1,A3d}{=} E[F^{-1}_{Y_1(0,1)|D=1,M=1} \circ F_{Y_0(0,1)|D=1,M=1}(Y_0(0,1))|D=1,M=1], \\
&\quad= E[Y_1(0,1)|D=1,M=1] = E[Y_1(0,1)|D=1,M(1)=1],
\end{aligned} \tag{A.40}$$

which has data support under Assumption 4d.

A.8 Quantile direct effect under d = 1 conditional on D = 1 and M(1) = 1

In the following, we show that

$$\begin{aligned}
\theta_1^{1,1}(q,1) &= F^{-1}_{Y_1(1,1)|D=1,M(1)=1}(q) - F^{-1}_{Y_1(0,1)|D=1,M(1)=1}(q), \\
&= F^{-1}_{Y_1|D=1,M=1}(q) - F^{-1}_{Q_{01}(Y_0)|D=1,M=1}(q).
\end{aligned}$$

For this purpose, we have to prove that

$$F_{Y_1(1,1)|D=1,M(1)=1}(y) = F_{Y_1|D=1,M=1}(y) \text{ and} \tag{A.41}$$

$$F_{Y_1(0,1)|D=1,M(1)=1}(y) = F_{Q_{01}(Y_0)|D=1,M=1}(y), \tag{A.42}$$

which is sufficient to show that the quantiles are also identified. We can show (A.41) using the observational rule $F_{Y_1(1,1)|D=1,M(1)=1}(y) = F_{Y_1|D=1,M=1}(y) = E[1\{Y_1 \leq y\}|D=1, M=1]$.

In analogy to (A.40), we obtain

$$\begin{aligned}
&F_{Q_{01}(Y_0)|D=1,M=1}(y) \\
&\quad = E[1\{Q_{01}(Y_0) \leq y\}|D=1, M=1], \\
&\quad = E[1\{F^{-1}_{Y_1|D=0,M=1} \circ F_{Y_0|D=0,M=1}(Y_0) \leq y\}|D=1, M=1], \\
&\quad = E[1\{Y_1(0,1) \leq y\}|D=1, M=0], \\
&\quad = F_{Y_1(0,1)|D=1,M(1)=1}(y),
\end{aligned}$$

which proves (A.42).

B Proof of Equations (1) and (2)

The average total effect for the entire population is identified by,

$$\begin{aligned}
\Delta_1 &= E[Y_1(1, M(1))] - E[Y_1(0, M(0))], \\
&\overset{A5}{=} E[Y_1(1, M(1))|D=1] - E[Y_1(0, M(0))|D=0], \\
&= E[Y_1|D=1] - E[Y_1|D=0],
\end{aligned}$$

where the first equality is the definition of Δ_1, the second equality hold by Assumption 5, and the last equality holds by the observational rule.

We define the conditional distribution $F_{Y_1|D=d}(y) = \Pr(Y_1 \leq y|D=d)$ and $F^{-1}_{Y_1|D=d}(q) = \inf\{y : F_{Y_1|D=d}(y) \geq q\}$. We can show the identification of the total QTE for the entire population $\Delta_1(q) = F^{-1}_{Y_1|D=1}(q) - F^{-1}_{Y_1|D=0}(q)$ when we show that

$F_{Y_1(1,M(1))}(y) = F_{Y_1|D=1}(y)$ and $F_{Y_1(0,M(0))}(y) = F_{Y_1|D=0}(y)$. Using Assumption 5 and the observational rule gives,

$$\begin{aligned} F_{Y_1(1,M(1))}(y) &= \Pr(Y_1(1, M(1)) \leq y), \\ &\overset{A5}{=} \Pr(Y_1(1, M(1)) \leq y|D = 1), \\ &= \Pr(Y_1 \leq y|D = 1) = F_{Y_1|D=1}(y), \end{aligned}$$

and

$$\begin{aligned} F_{Y_1(0,M(0))}(y) &= \Pr(Y_1(0, M(0)) \leq y), \\ &\overset{A5}{=} \Pr(Y_1(0, M(0)) \leq y|D = 0), \\ &= \Pr(Y_1 \leq y|D = 0) = F_{Y_1|D=0}(y), \end{aligned}$$

which finishes the proof.

By Assumption 5, the share of a type τ conditional on D corresponds to p_τ (in the population), as D is randomly assigned. This implies that $p_{1|1} = p_{n1} + p_{ap}$, $p_{1|0} = p_{n1} + p_{an}$, $p_{0|1} = p_{n0} + p_{an}$, and $p_{0|0} = p_{n0} + p_{ap}$. Under Assumption 6, $p_{an} = 0$, which finishes the proof of equation (1).

Furthermore, $E[Y_t(d, m)|\tau, D = 1] = E[Y_t(d, m)|\tau, D = 0] = E[Y_t(d, m)|\tau]$ due to the independence of D and the potential outcomes as well as the types τ (which are a deterministic function of $M(d)$) under Assumption 5. It follows that conditioning on D is not required on the right hand side of the following equation, which expresses the mean outcome conditional $D = 0$ and $M = 0$ as weighted average of the mean potential outcomes of affected positively and not-affected at 0:

$$\begin{aligned} &E[Y_t|D = 0, M = 0] \\ &\quad = \frac{p_{n0}}{p_{n0} + p_{ap}} E[Y_t(0, 0)|\tau = n0] + \frac{p_{ap}}{p_{n0} + p_{ap}} E[Y_t(0, 0)|\tau = ap]. \end{aligned} \tag{B.1}$$

Only affected positively and not-affected at 0 satisfy $M(0) = 0$ and thus make up the group with $D = 0$ and $M = 0$. After some rearrangements we obtain

$$\begin{aligned} &E[Y_t(0,0)|\tau = n0] - E[Y_t(0,0)|\tau = ap] \\ &\quad = \frac{p_{n0} + p_{ap}}{p_{ap}} \left\{ E[Y_t(0,0)|\tau = n0] - E[Y_t|D = 0, M = 0] \right\}. \end{aligned} \tag{B.2}$$

Next, we consider observations with $D = 1$ and $M = 0$, which might consist of both not-affected at 0 and affected negatively, as $M(1) = 0$ for both types. However, by Assumption 6, affected negatively are ruled out, such that the mean outcome given $D_1 = 1$ and $M_1 = 0$ is determined by not-affected at 0 only:

$$E[Y_t|D = 1, M = 0] \overset{A5,A6}{=} E[Y_t(1,0)|\tau = n0]. \tag{B.3}$$

Furthermore, by Assumption 2,

$$E[Y_0(0,0)|\tau = n0] \overset{A2}{=} E[Y_0(1,0)|\tau = n0] \overset{A5,A6}{=} E[Y_0|D = 1, M = 0].$$

Similarly to (B.1) for the not-affected at 0 and affected positively, consider the mean outcome given $D = 1$ and $M = 1$, which is made up by not-affected at 1 and affected positively (the types with $M(1) = 1$)

$$\begin{aligned} &E[Y_t|D = 1, M = 1] \\ &\quad = \frac{p_{n1}}{p_{n1} + p_{ap}} E[Y_t(1,1)|\tau = n1] + \frac{p_{ap}}{p_{n1} + p_{ap}} E[Y_t(1,1)|\tau = ap]. \end{aligned} \tag{B.4}$$

After some rearrangements we obtain

$$\begin{aligned} &E[Y_t(1,1)|\tau = n1] - E[Y_t(1,1)|\tau = ap] \\ &\quad = \frac{p_{n1} + p_{ap}}{p_{ap}} \left\{ E[Y_t(1,1)|\tau = n1] - E[Y_t|D = 1, M = 1] \right\}. \end{aligned} \tag{B.5}$$

By Assumptions 5 and 6,

$$E[Y_t|D = 0, M = 1] = E[Y_t(0,1)|\tau = n1]. \tag{B.6}$$

Now consider (B.5) for period $T = 0$, and note that by Assumption 2, $E[Y_0(1,1)|\tau = n1] = E[Y_0(0,0)|\tau = n1] = E[Y_0(0,1)|\tau = n1]$ and $E[Y_0(1,1)|\tau = ap] = E[Y_0(0,0)|\tau = ap]$.

Combining (B.4), (B.6), and the law of iterative expectations (LIE) gives

$$
\begin{aligned}
&E[Y_0|D=1] \\
&\quad \overset{LIE}{=} E[Y_0|D=1, M=1] \cdot p_{1|1} + E[Y_0|D=1, M=0] \cdot p_{0|1}, \\
&\quad = E[Y_0(1,1)|\tau = ap] \cdot p_{ap} + E[Y_0(1,1)|\tau = n1] \cdot p_{n1} + E[Y_0(1,0)|\tau = n0] \cdot p_{n0}, \\
&\quad \overset{A2}{=} E[Y_0(1,1)|\tau = ap] \cdot p_{ap} + E[Y_0(1,1)|\tau = n1] \cdot p_{n1} + E[Y_0(0,0)|\tau = n0] \cdot p_{n0}.
\end{aligned}
$$

Likewise, combining (B.1) and (B.3) gives

$$
\begin{aligned}
&E[Y_0|D=0] \\
&\quad \overset{LIE}{=} E[Y_0|D=0, M=1] \cdot p_{1|0} + E[Y_0|D_0=1, M=0] \cdot p_{0|0}, \\
&\quad = E[Y_0(0,1)|\tau = n1] \cdot p_{n1} + E[Y_0(0,0)|\tau = ap] \cdot p_{ap} + E[Y_0(0,0)|\tau = n0] \cdot p_{n0}, \\
&\quad \overset{A2}{=} E[Y_0(1,1)|\tau = n1] \cdot p_{n1} + E[Y_0(0,0)|\tau = ap] \cdot p_{ap} + E[Y_0(0,0)|\tau = n0] \cdot p_{n0}.
\end{aligned}
$$

Accordingly,

$$
\frac{E[Y_0|D=1] - E[Y_0|D=0]}{p_{1|1} - p_{1|0}} = E[Y_0(1,1)|\tau = ap] - E[Y_0(0,0)|\tau = ap] \overset{A2}{=} 0,
$$

which proves equation (2). Accordingly, $E[Y_0|D=1] - E[Y_0|D=0] = 0$ is a testable implication of Assumption 2, 5, and 6.

C Proof of Theorem 2

C.1 Average direct effect on the not-affected at 0

In the following, we show that $\theta_1^{n0} = E[Y_1(1,0) - Y_1(0,0)|\tau = n0] = E[Y_1 - Q_{00}(Y_0)|D = 1, M = 0]$. From (B.3), we obtain the first ingredient $E[Y_1(1,0)|\tau =$

$n0] = E[Y_1|D = 1, M = 0]$. Furthermore, from (A.9) we have $E[Q_{00}(Y_0)|D = 1, M = 0] = E[Y_1(0,0)|D = 1, M(1) = 0]$. Under Assumption 5 and 6,

$$E[Y_1(0,0)|D = 1, M(1) = 0] = E[Y_1(0,0)|D = 1, \tau = n0] = E[Y_1(0,0)|\tau = n0]. \tag{C.1}$$

C.2 Quantile direct effect on the not-affected at 0

We prove that

$$\begin{aligned}\theta_1^{n0}(q) &= F^{-1}_{Y_1(1,0)|n0}(q) - F^{-1}_{Y_1(0,0)|n0}(q),\\ &= F^{-1}_{Y_1|D=1,M=0}(q) - F^{-1}_{Q_{00}(Y_0)|D=1,M=0}(q).\end{aligned}$$

This requires showing that

$$F_{Y_1(1,0)|n0}(y) = F_{Y_1|D=1,M=0}(y) \text{ and} \tag{C.2}$$

$$F_{Y_1(0,0)|n0}(y) = F_{Q_{00}(Y_0)|D=1,M=0}(y). \tag{C.3}$$

Under Assumptions 5 and 6,

$$\begin{aligned}F_{Y_t|D=1,M=0}(y) &= E[1\{Y_t \le y\}|D = 1, M = 0]\\ &\overset{A5,A6}{=} E[1\{Y_t(1,0) \le y\}|\tau = n0]\\ &= F_{Y_t(1,0)|n0}(y),\end{aligned} \tag{C.4}$$

which proves (C.2). From (A.12), we have

$$F_{Q_{00}(Y_0)|D=1,M=0}(y) = F_{Y_1(0,0)|D=1,M(1)=0}(y) = E[1\{Y_1(0,0) \le y\}|D = 1, M(1) = 0].$$

Under Assumption 5 and 6,

$$\begin{aligned}E[1\{Y_1(0,0) \le y\}|D = 1, M(1) = 0] &\overset{A5,A6}{=} E[1\{Y_1(0,0) \le y\}|\tau = n0]\\ &= F_{Y_1(0,0)|n0}(y),\end{aligned} \tag{C.5}$$

which proves (C.3).

C.3 Average direct effect under d = 0 on affected positively

In the following, we show that

$$\begin{aligned}\theta_1^{ap}(0) =& E[Y_1(1,0) - Y_1(0,0)|\tau = ap], \\ =& \frac{p_{0|0}}{p_{0|0} - p_{0|1}} E[Q_{10}(Y_0) - Y_1|D = 0, M = 0] \\ &- \frac{p_{0|1}}{p_{0|0} - p_{0|1}} E[Y_1 - Q_{00}(Y_0)|D = 1, M = 0].\end{aligned}$$

Plugging (C.1) in (B.1) under $T = 1$, we obtain

$$\begin{aligned}E[Y_1|D = 0, M = 0] = & \frac{p_{n0}}{p_{n0} + p_{ap}} E[Q_{00}(Y_0)|D = 1, M = 0] \\ &+ \frac{p_{ap}}{p_{n0} + p_{ap}} E[Y_1(0,0)|\tau = ap].\end{aligned}$$

This allows identifying

$$\begin{aligned}E[Y_1(0,0)|\tau = ap] = & \frac{p_{0|0}}{p_{0|0} - p_{0|1}} E[Y_1|D = 0, M = 0] \\ &- \frac{p_{0|1}}{p_{0|0} - p_{0|1}} E[Q_{00}(Y_0)|D = 1, M = 0].\end{aligned} \tag{C.6}$$

Accordingly, we have to show the identification of $E[Y_1(1,0)|ap]$ to finish the proof. From (A.19) we have $E[Y_1(1,0)|D = 0, M = 0] = E[Q_{10}(Y_0)|D = 0, M = 0]$. Applying the law of iterative expectations, gives

$$\begin{aligned}E[Y_1(1,0)|D = 0, M = 0] =& \frac{p_{n0}}{p_{n0} + p_{ap}} E[Y_1(1,0)|D = 0, M = 0, \tau = n0] \\ &+ \frac{p_{ap}}{p_{n0} + p_{ap}} E[Y_1(1,0)|D = 0, M = 0, \tau = ap], \\ \overset{A5}{=}& \frac{p_{n0}}{p_{n0} + p_{ap}} E[Y_1(1,0)|\tau = n0] + \frac{p_{ap}}{p_{n0} + p_{ap}} E[Y_1(1,0)|\tau = ap].\end{aligned}$$

After some rearrangements and using (B.3), we obtain

$$E[Y_1(1,0)|\tau = ap] = \frac{p_{n0} + p_{ap}}{p_{ap}} E[Q_{10}(Y_0)|D = 0, M = 0] - \frac{p_{n0}}{p_{ap}} E[Y_1|D = 1, M = 0].$$

This gives

$$\begin{aligned} E[Y_1(1,0)|\tau = ap] = & \frac{p_{0|0}}{p_{0|0} - p_{0|1}} E[Q_{10}(Y_0)|D = 0, M = 0] \\ & - \frac{p_{0|1}}{p_{0|0} - p_{0|1}} E[Y_1|D = 1, M = 0], \end{aligned} \tag{C.7}$$

using $p_{n0} = p_{0|1}$, and $p_{ap} + p_{n0} = p_{0|0}$.

C.4 Quantile direct effect under d = 0 on affected positively

We show that

$$F_{Y_1(1,0)|ap}(y) = \frac{p_{0|0}}{p_{0|0} - p_{0|1}} F_{Q_{10}(Y_0)|D=0,M=0}(y) - \frac{p_{0|1}}{p_{0|0} - p_{0|1}} F_{Y_1|D=1,M=0}(y) \text{ and}$$
$$F_{Y_1(0,0)|ap}(y) = \frac{p_{0|0}}{p_{0|0} - p_{0|1}} F_{Y_1|D=0,M=0}(y) - \frac{p_{0|1}}{p_{0|0} - p_{0|1}} F_{Q_{00}(Y_0)|D=1,M=0}(y),$$

which proves that $\theta_1^{ap}(q, 0) = F^{-1}_{Y_1(1,0)|ap}(q) - F^{-1}_{Y_1(0,0)|ap}(q)$ is identified.

From (A.20), we have $F_{Y_1(1,0)|D=0,M(0)=0}(y) = F_{Q_{10}(Y_0)|D=0,M=0}(y)$. Applying the law of iterative expectations gives

$$\begin{aligned} F_{Y_1(1,0)|D=0,M(0)=0}(y) = & \frac{p_{n0}}{p_{n0} + p_{ap}} F_{Y_1(1,0)|D=0,M(0)=0,\tau=n0}(y) \\ & + \frac{p_{ap}}{p_{n0} + p_{ap}} F_{Y_1(1,0)|D=0,M(0)=0,\tau=ap}(y), \\ \overset{A5}{=} & \frac{p_{n0}}{p_{n0} + p_{ap}} F_{Y_1(1,0)|n0}(y) + \frac{p_{ap}}{p_{n0} + p_{ap}} F_{Y_1(1,0)|ap}(y). \end{aligned}$$

Using (C.2) and rearranging the equation gives,

$$F_{Y_1(1,0)|ap}(y) = \frac{p_{0|0}}{p_{0|0} - p_{0|1}} F_{Q_{10}(Y_0)|D=0,M=0}(y) - \frac{p_{0|1}}{p_{0|0} - p_{0|1}} F_{Y_1|D=1,M=0}(y). \tag{C.8}$$

In analogy to (B.1), the outcome distribution under $D=0$ and $M=0$ equals

$$F_{Y_1|D=0,M=0}(y)=\frac{p_{n0}}{p_{n0}+p_{ap}}F_{Y_1(0,0)|n0}(y)+\frac{p_{ap}}{p_{n0}+p_{ap}}F_{Y_1(0,0)|ap}(y).$$

Using (C.3) and rearranging the equation gives

$$F_{Y_1(0,0)|ap}(y)=\frac{p_{0|0}}{p_{0|0}-p_{0|1}}F_{Y_1|D=0,M=0}(y)-\frac{p_{0|1}}{p_{0|0}-p_{0|1}}F_{Q_{00}(Y_0)|D=1,M=0}(y). \quad \text{(C.9)}$$

C.5 Average direct effect on the not-affected at 1

In the following, we show that $\theta_1^{n1}=E[Y_1(1,1)-Y_1(0,1)|\tau=n1]=E[Q_{11}(Y_0)-Y_1|D=0,M=1]$. From (B.6), we obtain the first ingredient $E[Y_1(0,1)|n1]=E[Y_1|D=0,M=1]$. Furthermore, from (A.30) we have $E[Q_{11}(Y_0)|D=0,M=1]=E[Y_1(1,1)|D=0,M(0)=1]$. Under Assumption 5 and 6,

$$E[Y_1(1,1)|D=0,M(0)=1]=E[Y_1(1,1)|D=0,\tau=n1]=E[Y_1(1,1)|\tau=n1]. \quad \text{(C.10)}$$

C.6 Quantile direct effect on the not-affected at 1

We prove that

$$\begin{aligned}\theta_1^{n1}(q)&=F^{-1}_{Y_1(1,1)|n1}(q)-F^{-1}_{Y_1(0,1)|n1}(q),\\&=F^{-1}_{Q_{11}(Y_0)|D=0,M=1}(q)-F^{-1}_{Y_1|D=0,M=1}(q).\end{aligned}$$

This requires showing that

$$F_{Y_1(1,1)|n1}(y)=F_{Q_{11}(Y_0)|D=0,M=1}(y) \text{ and} \quad \text{(C.11)}$$

$$F_{Y_1(0,1)|n1}(y)=F_{Y_1|D=0,M=1}(y). \quad \text{(C.12)}$$

Under Assumptions 5 and 6,

$$
\begin{aligned}
F_{Y_t|D=0,M=1}(y) &= E[1\{Y_t \leq y\}|D=0, M=1] \\
&\overset{A5,A6}{=} E[1\{Y_t(0,1) \leq y\}|\tau = n1] \\
&= F_{Y_t(0,1)|n1}, (y).
\end{aligned} \tag{C.13}
$$

which proves (C.12). From (A.33), we have

$$
F_{Q_{11}(Y_0)|D=0,M=1}(y) = F_{Y_1(1,1)|D=0,M(0)=1}(y) = E[1\{Y_1(1,1) \leq y\}|D=0, M(0)=1].
$$

Under Assumption 5 and 6,

$$
\begin{aligned}
E[1\{Y_1(1,1) \leq y\}|D=0, M(0)=1] &\overset{A5,A6}{=} E[1\{Y_1(1,1) \leq y\}|\tau = n1] \\
&= F_{Y_1(1,1)|n1}(y),
\end{aligned} \tag{C.14}
$$

which proves (C.11).

C.7 Average direct effect under d = 1 on affected positively

In the following, we show that

$$
\begin{aligned}
\theta_1^{ap}(1) =& E[Y_1(1,1) - Y_1(0,1)|\tau = ap], \\
=& \frac{p_{1|1}}{p_{1|1} - p_{1|0}} E[Y_1 - Q_{01}(Y_0)|D=1, M=1] \\
&- \frac{p_{1|0}}{p_{1|1} - p_{1|0}} E[Q_{11}(Y_0) - Y_1|D=0, M=1].
\end{aligned}
$$

Plugging (C.10) in (B.4), we obtain

$$
\begin{aligned}
E[Y_1|D=1, M=1] = & \frac{p_{n1}}{p_{n1} + p_{ap}} E[Q_{11}(Y_0)|D=0, M=1] \\
&+ \frac{p_{ap}}{p_{n1} + p_{ap}} E[Y_1(1,1)|\tau = ap].
\end{aligned}
$$

This allows identifying

$$E[Y_1(1,1)|\tau = ap] = \frac{p_{1|1}}{p_{1|1} - p_{1|0}} E[Y_1|D = 1, M = 1] - \frac{p_{1|0}}{p_{1|1} - p_{1|0}} E[Q_{11}(Y_0)|D = 0, M = 1]. \tag{C.15}$$

From (A.40) we have $E[Y_1(0,1)|D = 1, M = 1] = E[Q_{01}(Y_0)|D = 1, M = 1]$. Applying the law of iterative expectations, gives

$$\begin{aligned} E[Y_1(0,1)|D = 1, M = 1] =& \frac{p_{n1}}{p_{n1} + p_{ap}} E[Y_1(0,1)|D = 1, M = 1, \tau = n1] \\ &+ \frac{p_{ap}}{p_{n1} + p_{ap}} E[Y_1(0,1)|D = 1, M = 1, \tau = ap], \\ \overset{A5}{=}& \frac{p_{n1}}{p_{n1} + p_{ap}} E[Y_1(0,1)|\tau = n1] + \frac{p_{ap}}{p_{n1} + p_{ap}} E[Y_1(0,1)|\tau = ap]. \end{aligned}$$

After some rearrangements and using (B.6), we obtain

$$E[Y_1(0,1)|\tau = ap] = \frac{p_{n1} + p_{ap}}{p_{ap}} E[Q_{01}(Y_0)|D = 1, M = 1] - \frac{p_{n1}}{p_{ap}} E[Y_1|D = 0, M = 1].$$

This gives

$$E[Y_1(0,1)|\tau = ap] = \frac{p_{1|1}}{p_{1|1} - p_{1|0}} E[Q_{01}(Y_0)|D = 1, M = 1] - \frac{p_{1|0}}{p_{1|1} - p_{1|0}} E[Y_1|D = 0, M = 1], \tag{C.16}$$

with $p_{n1} = p_{1|0}$, and $p_{ap} + p_{n1} = p_{1|1}$.

C.8 Quantile direct effect under d = 1 on affected positively

We show that

$$F_{Y_1(1,1)|ap}(y) = \frac{p_{1|1}}{p_{1|1} - p_{1|0}} F_{Y_1|D=1,M=1}(y) - \frac{p_{1|0}}{p_{1|1} - p_{1|0}} F_{Q_{11}(Y_0)|D=0,M=1}(y) \text{ and}$$
$$F_{Y_1(0,1)|ap}(y) = \frac{p_{1|1}}{p_{1|1} - p_{1|0}} F_{Q_{01}(Y_0)|D=1,M=1}(y) - \frac{p_{1|0}}{p_{1|1} - p_{1|0}} F_{Y_1|D=0,M=1}(y),$$

which proves that $\theta_1^{ap}(q,1) = F^{-1}_{Y_1(1,1)|ap}(q) - F^{-1}_{Y_1(0,1)|ap}(q)$ is identified.

In analogy to (B.4), the outcome distribution under $D = 0$ and $M = 0$ equals:

$$F_{Y_1|D=1,M=1}(y) = \frac{p_{n1}}{p_{n1} + p_{ap}} F_{Y_1(1,1)|n1}(y) + \frac{p_{ap}}{p_{n1} + p_{ap}} F_{Y_1(1,1)|ap}(y).$$

Using (C.11) and rearranging the equation gives

$$F_{Y_1(1,1)|ap}(y) = \frac{p_{1|1}}{p_{1|1} - p_{1|0}} F_{Y_1|D=1,M=1}(y) - \frac{p_{1|0}}{p_{1|1} - p_{1|0}} F_{Q_{11}(Y_0)|D=0,M=1}(y). \quad \text{(C.17)}$$

From (A.42), we have $F_{Y_1(0,1)|D=1,M(1)=1}(y) = F_{Q_{01}(Y_0)|D=1,M=1}(y)$. Applying the law of iterative expectations gives

$$\begin{aligned} F_{Y_1(0,1)|D=1,M(1)=1}(y) =& \frac{p_{n1}}{p_{n1} + p_{ap}} F_{Y_1(0,1)|D=1,M(1)=1,\tau=n1}(y) \\ &+ \frac{p_{ap}}{p_{n1} + p_{ap}} F_{Y_1(0,1)|D=1,M(1)=1,\tau=ap}(y), \\ \overset{A5}{=}& \frac{p_{n1}}{p_{n1} + p_{ap}} F_{Y_1(0,1)|n1}(y) + \frac{p_{ap}}{p_{n1} + p_{ap}} F_{Y_1(0,1)|ap}(y). \end{aligned}$$

Using (C.12) and rearranging the equation gives,

$$F_{Y_1(0,1)|ap}(y) = \frac{p_{1|1}}{p_{1|1} - p_{1|0}} F_{Q_{01}(Y_0)|D=1,M=1}(y) - \frac{p_{1|0}}{p_{1|1} - p_{1|0}} F_{Y_1|D=0,M=1}(y). \quad \text{(C.18)}$$

D Proof of Theorem 3

D.1 Average treatment effect on the affected positively

In (C.15) and (C.6), we show that

$$\begin{aligned} \theta_1^{ap} =& E[Y_1(1,1) - Y_1(0,0)|\tau = ap], \\ =& \frac{p_{1|1}}{p_{1|1} - p_{1|0}} E[Y_1|D = 1, M = 1] - \frac{p_{1|0}}{p_{1|1} - p_{1|0}} E[Q_{11}(Y_0)|D = 0, M = 1] \\ &- \frac{p_{0|0}}{p_{0|0} - p_{0|1}} E[Y_1|D = 0, M = 0] + \frac{p_{0|1}}{p_{0|0} - p_{0|1}} E[Q_{00}(Y_0)|D = 1, M = 0]. \end{aligned}$$

D.2 Quantile treatment effect on the affected positively

In (C.17) and (C.9), we show that $F_{Y_1(1,1)|ap}(y)$ and $F_{Y_1(0,0)|ap}(y)$ are identified. Accordingly, $\Delta_1^{ap}(q) = F^{-1}_{Y_1(1,1)|ap}(q) - F^{-1}_{Y_1(0,0)|ap}(q)$ is identified.

D.3 Average indirect effect under $d = 0$ on affected positively

In (C.16) and (C.6), we show that

$$\begin{aligned}\delta_1^{ap}(0) =& E[Y_1(0,1) - Y_1(0,0)|\tau = ap],\\ =& \frac{p_{1|1}}{p_{1|1} - p_{1|0}} E[Q_{01}(Y_0)|D = 1, M = 1] - \frac{p_{1|0}}{p_{1|1} - p_{1|0}} E[Y_1|D = 0, M = 1]\\ &- \frac{p_{0|0}}{p_{0|0} - p_{0|1}} E[Y_1|D = 0, M = 0] + \frac{p_{0|1}}{p_{0|0} - p_{0|1}} E[Q_{00}(Y_0)|D = 1, M = 0].\end{aligned}$$

D.4 Quantile indirect effect under $d = 0$ on affected positively

In (C.18) and (C.9), we show that $F_{Y_1(0,1)|ap}(y)$ and $F_{Y_1(0,0)|ap}(y)$ are identified. Accordingly, $\delta_1^{ap}(q, 0) = F^{-1}_{Y_1(0,1)|ap}(q) - F^{-1}_{Y_1(0,0)|ap}(q)$ is identified.

D.5 Average indirect effect under $d = 1$ on affected positively

In (C.15) and (C.7), we show that

$$\begin{aligned}\delta_1^{ap}(1) =& E[Y_1(1,1) - Y_1(1,0)|\tau = ap],\\ =& \frac{p_{1|1}}{p_{1|1} - p_{1|0}} E[Y_1|D = 1, M = 1] - \frac{p_{1|0}}{p_{1|1} - p_{1|0}} E[Q_{11}(Y_0)|D = 0, M = 1]\\ &- \frac{p_{0|0}}{p_{0|0} - p_{0|1}} E[Q_{10}(Y_0)|D = 0, M = 0] + \frac{p_{0|1}}{p_{0|0} - p_{0|1}} E[Y_1|D = 1, M = 0].\end{aligned}$$

D.6 Quantile indirect effect under $d = 1$ on affected positively

In (C.17) and (C.8), we show that $F_{Y_1(1,1)|ap}(y)$ and $F_{Y_1(1,0)|ap}(y)$ are identified. Accordingly, $\delta_1^{ap}(q,1) = F^{-1}_{Y_1(1,1)|ap}(q) - F^{-1}_{Y_1(1,0)|ap}(q)$ is identified.

E Simulation study

To shape the intuition for our identification results, this appendix presents a brief simulation based on the following data generating process (DGP):

$$T \sim Binom(0.5),\ D \sim Binom(0.5),\ U \sim Unif(-1,1),\ V \sim N(0,1)$$

independent of each other, and

$$M = I\{D + U + V > 0\}, \qquad Y_T = \Lambda((1 + D + M + D \cdot M) \cdot T + U).$$

Treatment D as well as the observed time period T are randomized and binomially distributed with a 50% chance of being 1 or 0, while the mediator-outcome association is confounded due to the unobserved time constant heterogeneity U (implying $U_0 = U_1$). The potential outcome in period 1 is given by $Y_1(d, M(d')) = \Lambda((1 + d + M(d') + d \cdot M(d')) + U)$, where Λ denotes a link function. If the latter corresponds to the identity function, our model is linear and implies a homogeneous time trend T equal to 1. If Λ is nonlinear, the time trend is heterogeneous, which invalidates the common trend assumption of DiD models. M is not only a function of D and U, but also of the unobserved random term V, which guarantees common support w.r.t. U, see Assumption 4. Affected positively, not-affected at 1, and not-affected at 0 satisfy, respectively: $ap = I\{U + V \leq 0, 1 + U + V > 0\}$, $n1 = I\{U + V > 0\}$, and $n0 = I\{1 + U + V \leq 0\}$.

In the simulations with 1,000 replications, we consider two sample sizes ($N = 1,000, 4,000$) and investigate the behaviour of our CiC approach as well as the

DiD approach of Deuchert, Huber, and Schelker (2019) in both a linear (Λ equal to identity function) and nonlinear outcome model where Λ equals the exponential function. The latter implies that a specific ceteris paribus change in a right hand variable entails a specific percentage change in the outcome (rather than a specific level change as in the linear model). To implement the CiC estimators in the simulations as well as the application in Section 4, we make use of the 'cic' command in the `qte` R-package by Callaway (2016) with its default values.

Table E.1: Linear model with random treatment

	$\hat{\theta}_1^{n0}$	$\hat{\theta}_1^{n1}$	$\hat{\Delta}_1^{ap}$	$\hat{\theta}_1^{ap}(1)$	$\hat{\theta}_1^{ap}(0)$	$\hat{\delta}_1^{ap}(1)$	$\hat{\delta}_1^{ap}(0)$
A. Changes-in-Changes							
N=1,000							
bias	0.00	-0.00	-0.01	-0.01	-0.01	-0.00	-0.01
sd	0.11	0.08	0.23	0.10	0.13	0.27	0.27
rmse	0.11	0.08	0.23	0.10	0.13	0.27	0.27
true	1.00	2.00	3.00	2.00	1.00	2.00	1.00
relr	0.11	0.04	0.08	0.05	0.13	0.14	0.27
N=4,000							
bias	-0.00	-0.00	0.00	-0.00	-0.01	0.01	0.01
sd	0.06	0.04	0.12	0.05	0.07	0.14	0.14
rmse	0.06	0.04	0.12	0.05	0.07	0.14	0.14
true	1.00	2.00	3.00	2.00	1.00	2.00	1.00
relr	0.06	0.02	0.04	0.02	0.07	0.07	0.14
B. Difference-in-Differences							
N=1,000							
bias	0.01	-0.00	-0.01	-0.01	0.00	-0.02	0.00
sd	0.11	0.09	0.14	0.14	0.12	0.19	0.10
rmse	0.11	0.09	0.14	0.14	0.12	0.19	0.10
true	1.00	2.00	3.00	2.00	1.00	2.00	1.00
relr	0.11	0.04	0.05	0.07	0.12	0.10	0.10
N=4,000							
bias	-0.00	-0.00	0.00	-0.00	-0.00	0.00	0.00
sd	0.06	0.04	0.07	0.07	0.06	0.10	0.05
rmse	0.06	0.04	0.07	0.07	0.06	0.10	0.05
true	1.00	2.00	3.00	2.00	1.00	2.00	1.00
relr	0.06	0.02	0.02	0.04	0.06	0.05	0.05

Note: 'bias', 'sd', and 'rmse' provide the bias, standard deviation, and root mean squared error of the respective estimator. 'true' and 'relr' are the respective true effect as well as the root mean squared error relative to the true effect.

Table E.1 reports the bias, standard deviation ('sd'), root mean squared error ('rmse'), true effect ('true'), and the relative root mean squared error in percent of the true effect ('relr') of the respective estimators of θ_1^{n0}, θ_1^{n1}, Δ_1^{ap}, $\theta_1^{ap}(1)$, $\theta_1^{ap}(0)$, $\delta_1^{ap}(1)$, and $\delta_1^{ap}(0)$ for the linear model. In this case, the identifying assumptions underlying both the CiC (Panel A.) and DiD (Panel B.) estimators are satisfied. Specifically, the homogeneous time trend on the cross-sectional observation unit satisfies any of the common trend assumptions in Deuchert, Huber, and Schelker (2019), while the monotonicity of Y in U and the independence of T and U satisfies the key assumptions of this paper. For this reason any of the estimates in Table E.1 are close to being unbiased and appear to converge to the true effect at the parametric rate when comparing the results for the two different sample sizes.[1]

Table E.2 provides the results for the exponential outcome model, in which the time trend is heterogeneous and interacts with U through the nonlinear link function. While the CiC assumptions hold (Panel A.), average time trends are heterogeneous across complier types such that the DiD approach (Panel B.) of Deuchert, Huber, and Schelker (2019) is inconsistent. Accordingly, the biases of the CiC estimates generally approach zero as the sample size increases, while this is not the case for the DiD estimates. CiC yields a lower root mean squared error than the respective DiD estimator in all but one case (namely $\hat{\delta}_1^{ap}(0)$ with $N = 1,000$) and its relative attractiveness increases in the sample size due to its lower bias.[2]

In our next simulation design, we maintain the exponential outcome model but assume D to be selective w.r.t. U rather than random. To this end, the treatment model in (E) is replaced by $D = I\{U + Q > 0\}$, with the independent variable $Q \sim N(0,1)$ being an unobserved term. The average of U among the treated and no-treated amounts to 0.24 and -0.24, respectively. This treatment selectivity entails

[1]In contrast, two stage least squares regression using D as instrument for M is inconsistent due to the direct effects violating the IV exclusion restriction. The IV estimate neither recovers Δ_1^{ap}, nor $\delta_1^{ap}(1)$, nor $\delta_1^{ap}(0)$, with the bias amounting to approximately 4, 5, and 6, respectively, for the three parameters with the sample sizes considered. This motivates the application of our method to verify the IV exclusion restriction in Section 4.

[2]However, we can easily modify the DGP underlying Table E.2 to match a scenario in which also CiC is inconsistent, e.g. by a violation of Assumption 3. For instance, when changing the distribution of U to $U|T = 0 \sim Unif(-1,1)$ and $U|T = 1 \sim Unif(0,1)$ such that it depends on T, we obtain non-negligible biases in the CiC estimates that do not vanish as the sample size increases.

Table E.2: Nonlinear model with random treatment

	$\hat{\theta}_1^{n0}$	$\hat{\theta}_1^{n1}$	$\hat{\Delta}_1^{ap}$	$\hat{\theta}_1^{ap}(1)$	$\hat{\theta}_1^{ap}(0)$	$\hat{\delta}_1^{ap}(1)$	$\hat{\delta}_1^{ap}(0)$
A. Changes-in-Changes							
N=1,000							
bias	0.01	-0.14	-0.48	-0.35	-0.11	-0.37	-0.13
sd	0.48	5.08	8.47	6.20	1.16	8.64	4.23
rmse	0.48	5.08	8.48	6.21	1.17	8.65	4.23
true	3.49	68.09	52.42	47.70	4.72	47.70	4.72
relr	0.14	0.07	0.16	0.13	0.25	0.18	0.90
N=4,000							
bias	-0.01	0.01	-0.00	-0.11	-0.07	0.07	0.11
sd	0.25	2.63	4.37	3.20	0.66	4.44	2.04
rmse	0.25	2.63	4.37	3.20	0.66	4.44	2.04
true	3.49	68.09	52.45	47.73	4.72	47.73	4.72
relr	0.07	0.04	0.08	0.07	0.14	0.09	0.43
B. Difference-in-Differences							
N=1,000							
bias	-0.27	-8.91	14.42	11.46	-1.49	15.91	2.96
sd	0.46	2.62	2.58	2.62	0.47	2.61	0.47
rmse	0.53	9.29	14.65	11.76	1.56	16.12	2.99
true	3.49	68.09	52.42	47.70	4.72	47.70	4.72
relr	0.15	0.14	0.28	0.25	0.33	0.34	0.63
N=4,000							
bias	-0.28	-8.79	14.51	11.57	-1.51	16.02	2.94
sd	0.24	1.28	1.26	1.28	0.25	1.27	0.23
rmse	0.37	8.88	14.57	11.64	1.53	16.07	2.95
true	3.49	68.09	52.45	47.73	4.72	47.73	4.72
relr	0.11	0.13	0.28	0.24	0.32	0.34	0.62

Note: 'bias', 'sd', and 'rmse' provide the bias, standard deviation, and root mean squared error of the respective estimator. 'true' and 'relr' are the respective true effect as well as the root mean squared error relative to the true effect.

non-negligible differences in mean potential outcomes across treatment groups, e.g. $E[Y_1(1,1)|D=1] - E[Y_1(1,1)|D=0] = 29.1$. Under this violation of Assumption 5, the shares and effects of affected positively are no longer identified, which is confirmed by the simulation results presented in Table E.3. The bias in the CiC based total, direct, and indirect effects on affected positively do not vanish as the sample size increases. Furthermore, under non-random assignment of D (while maintaining monotonicity of M in D), the not-affected at 0 and 1 respective distributions of U differ across treatment. Therefore, average direct effects among the total of not-

Table E.3: Nonlinear model with non-random treatment

	$\hat{\theta}_1^{0,1}(1)$	$\hat{\theta}_1^{1,0}(0)$	$\hat{\Delta}_1^{ap}$	$\hat{\theta}_1^{ap}(1)$	$\hat{\theta}_1^{ap}(0)$	$\hat{\delta}_1^{ap}(1)$	$\hat{\delta}_1^{ap}(0)$
A. Changes-in-Changes							
N=1,000							
bias	0.02	0.13	47.21	40.19	-1.44	48.64	7.02
sd	0.71	4.56	5.45	4.11	0.75	5.53	2.92
rmse	0.71	4.56	47.52	40.40	1.62	48.96	7.60
true	4.41	54.19	52.42	47.70	4.72	47.70	4.72
relr	0.16	0.08	0.91	0.85	0.34	1.03	1.61
N=4,000							
bias	-0.00	0.06	47.38	40.13	-1.53	48.91	7.25
sd	0.38	2.35	2.84	2.04	0.38	2.86	1.51
rmse	0.38	2.35	47.47	40.18	1.57	48.99	7.40
true	4.40	54.18	52.45	47.73	4.72	47.73	4.72
relr	0.09	0.04	0.90	0.84	0.33	1.03	1.57
B. Difference-in-Differences							
N=1,000							
bias	0.35	19.98	29.00	27.65	0.04	28.96	1.35
sd	0.67	2.48	2.46	2.48	0.67	2.51	0.45
rmse	0.75	20.14	29.11	27.76	0.67	29.07	1.43
true	4.41	54.19	52.42	47.70	4.72	47.70	4.72
relr	0.17	0.37	0.56	0.58	0.14	0.61	0.30
N=4,000							
bias	0.34	20.02	28.98	27.65	0.02	28.96	1.33
sd	0.35	1.22	1.19	1.22	0.35	1.24	0.23
rmse	0.49	20.06	29.01	27.68	0.35	28.99	1.35
true	4.40	54.18	52.45	47.73	4.72	47.73	4.72
relr	0.11	0.37	0.55	0.58	0.07	0.61	0.29

Note: 'bias', 'sd', and 'rmse' provide the bias, standard deviation, and root mean squared error of the respective estimator. 'true' and 'relr' are the respective true effect as well as the root mean squared error relative to the true effect.

affected at 0 or 1, respectively, are not identified. Yet, $\theta_1^{1,0}(1)$, which is still identified by the same estimator as before, yields the direct effect among treated not-affected at 0 (as affected negatively do not exist). Likewise, $\theta_1^{0,1}(0)$ corresponds to the direct effect on non-treated not-affected at 1. Indeed, the results in Table E.3 suggest that both parameters are consistently estimated by the CiC approach (Panel A.).

Finally, we also consider a violation of Assumption 6 by relaxing monotonicity of M in D. We do so by modifying the mediator equation to $M = I\{(2\kappa - 1) \cdot D + U + V > 0\}$, with $\kappa \sim Binom(0.2)$ being a randomly and binomially distributed variable, implying that the coefficient on D is either 1 or -1 with a probability of 80% or 20%, respectively. This entails a defier share of roughly 9% in the population, while we otherwise maintain the specification underlying the results in Table E.3. We note that $\theta_1^{1,0}(1)$ now corresponds to the direct effect on treated not-affected at 0 and affected negatively, $\theta_1^{0,1}(0)$ on non-treated not-affected at 1 and affected negatively. Table E.4 provides the results. Again, CiC performs decently for estimating $\theta_1^{1,0}(1)$ and $\theta_1^{0,1}(0)$ as suggested by Theorem 1, while non-negligible relative root mean squared error arise for the remaining parameters.

Table E.4: Nonlinear model with non-random treatment and non-monotonicity

	$\hat{\theta}_1^{0,1}(1)$	$\hat{\theta}_1^{1,0}(0)$	$\hat{\Delta}_1^{ap}$	$\hat{\theta}_1^{ap}(1)$	$\hat{\theta}_1^{ap}(0)$	$\hat{\delta}_1^{ap}(1)$	$\hat{\delta}_1^{ap}(0)$
A. Changes-in-Changes							
N=1,000							
bias	0.06	0.24	65.65	55.29	-3.76	69.41	10.35
stdev	0.62	4.90	10.98	7.74	0.86	11.25	6.47
rmse	0.62	4.91	66.56	55.83	3.86	70.31	12.21
true	5.62	54.19	52.45	47.73	4.72	47.73	4.72
relr	0.11	0.09	1.27	1.17	0.82	1.47	2.59
N=4,000							
bias	0.02	0.10	65.91	55.01	-3.84	69.75	10.90
stdev	0.31	2.49	5.80	4.03	0.46	5.99	3.23
rmse	0.32	2.49	66.17	55.16	3.86	70.00	11.36
true	5.63	54.18	52.45	47.73	4.72	47.73	4.72
relr	0.06	0.05	1.26	1.16	0.82	1.47	2.41
B. Difference-in-Differences							
N=1,000							
bias	0.79	21.78	31.59	30.24	1.70	29.90	1.36
stdev	0.54	2.82	2.79	2.81	0.56	2.82	0.46
rmse	0.96	21.97	31.72	30.37	1.79	30.03	1.43
true	5.62	54.19	52.45	47.73	4.72	47.73	4.72
relr	0.17	0.41	0.60	0.64	0.38	0.63	0.30
N=4,000							
bias	0.80	21.76	31.54	30.21	1.70	29.84	1.33
stdev	0.27	1.36	1.33	1.36	0.28	1.35	0.24
rmse	0.84	21.80	31.57	30.24	1.72	29.87	1.35
true	5.63	54.18	52.45	47.73	4.72	47.73	4.72
relr	0.15	0.40	0.60	0.63	0.37	0.63	0.29

Note: 'bias', 'sd', and 'rmse' provide the bias, standard deviation, and root mean squared error of the respective estimator. 'true' and 'relr' are the respective true effect as well as the root mean squared error relative to the true effect.

F Background Information for Applications

F.1 JOBS II Evaluation

The JOBS II was a modified version of the earlier JOBS programme, which had been found to improve labour market outcomes such as job satisfaction, motivation, earnings, and job stability, see Caplan, Vinokur, Price, and van Ryn (1989) and Vinokur, van Ryn, Gramlich, and Price (1991), as well as mental health, see Vinokur, Price, and Caplan (1991). According to the results of Vinokur, Price, and Schul (1995), the JOBS II programme increased re-employment rates and improved mental health outcomes, especially for participants having an elevated risk of depression. The JOBS interventions had an important impact in the academic literature (see e.g. Wanberg, 2012, Liu, Huang, and Wang, 2014) and the methodology was implemented in field experiments in Finland (Vuori, Silvonen, Vinokur, and Price, 2002, Vuori and Silvonen, 2005) and the Netherlands (Brenninkmeijer and Blonk, 2011), suggesting positive effects on labour market integration in either case. Imai, Keele, and Tingley (2010) analyse Jobs II in a mediation context as well, but consider a different mediator, namely job search self-efficacy, and a different identification strategy based on selection on observables.

In the JOBS II intervention, individuals responded to a screening questionnaire that collected pre-treatment information on mental health in the baseline period. Based on the latter, individuals were classified as having either a high or low depression risk and those with a high risk were oversampled before the training was randomly assigned. Randomization was followed by yet another questionnaire sent out two weeks before the actual job training, see Vinokur, Price, and Schul (1995), which also provided information on whether an individual had been assigned the training. Consequently, the data collected in that questionnaire must be considered post-treatment as they could be affected by learning the assignment. Therefore, we rely on the earlier screening data as the relevant pre-treatment period prior to random programme assignment.

The job training consisted of five 4-hours seminars conducted in morning sessions during one week between March 1 and August 7, 1991. Members of the treatment group who participated in at least four of the five sessions received USD 20. Each of the standardized training sessions consisted, among other aspects, of the learning and practicing of job search and problem-solving skills. The control group received a booklet with information on job search methods (Vinokur, Price, and Schul, 1995, p. 44-49).

F.2 Paid Maternal Leave Reform

There is a large literature on the impact of maternal or parental leave on female labour supply, earnings, or fertility, see for instance Lalive and Zweimüller (2009), Lalive, Schlosser, Steinhauer, and Zweimüller (2014), Fitzenberger, Steffes, and Strittmatter (2016), Byker (2016), Dahl, Løken, Mogstad, and Salvanes (2016), Olivetti and Petrongolo (2017), and Zimmert and Zimmert (2020). The design of maternal or paternal leave programs varies substantially across countries and estimation results depend heavily on the design of such programs with respect to the leave duration, the income replacement rate, job protection regulation, the availability of paid leave to either parent, etc. (Olivetti and Petrongolo, 2017).

In Switzerland, paid maternal leave was only introduced in 2005. Before, the Law on Manufacturing of 1877 just prohibited maternal labour supply for 8 weeks, with at least 6 weeks taken right after childbirth. In 1945 the constitutional bases for a paid maternal leave were established. However, numerous attempts to actually introduce paid maternal leave were all rejected in nation-wide popular ballots, the last unsuccessful attempt only dating back to 1999. Finally, on September 24, 2004, a majority of 55.4% of Swiss citizens voted in favour of the introduction of 14 weeks of paid maternal leave, with a replacement rate of 80% and a cap at CHF 172 per day in 2005. Paid maternal leave is covered through the Swiss fund for loss of earnings and maternal pay. The reform took effect on July 1, 2005. Job protection regulation remained unaffected and protection against dismissal lasts during the entire pregnancy and 16 weeks after childbirth.

The political campaigning and discussions on the various topics on the agenda (there were four federal propositions in September) typically start two to three months before. Given that all previous attempts to introduce paid maternal leave were rejected in popular ballots, the latest in 1999, and that the subsequent acceptance with 55.4% was far from overwhelming, important anticipation effects are fairly unlikely. The post-treatment period contains information from the 2007 questionnaire. We do not use data from 2005 and 2006 because interviews of the Swiss Labour Force Survey are only conducted up to the end of June each year. This makes 2005 a pre-treatment period and childbirth in early 2006 is the result of fertility decisions before or just around the introduction of paid maternal leave legislation in July 2005.

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